MAGNETICS

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1 Sisällys

1 Magnetic and electric circuit analogy

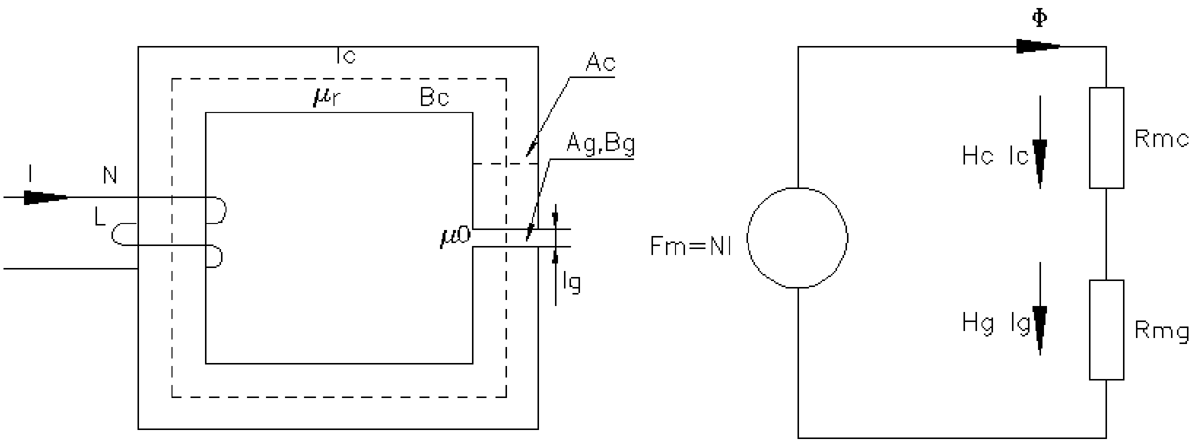


Figure 1:Magnetic circuit

I= current in winding (A)
F_m = Magnetomotive force (A)
N = Number of turns (unitless)
L = Inductance of winding (H)
μ_0 = Permeability of vacuum, $4\pi*10^{-7}$ (H/m)
μ_r = Relative permeability of material (unitless)
R_{mc} = Reluctance of the magnetic circuit in core (A/Vs=1/H)
R_{mg} = Reluctance of the magnetic circuit in air gap (A/Vs= 1/H)
lc = Length of magnetic circuit in core (m)
lg = Length of magnetic circuit in air gap (m)
A_c= Cross section of magnetic core (m^2)
A_g= Cross section of air gap (m^2)
V_c = Core volume (m^3)
V_g = Air gap volume (m^3)
B_c = Flux density in core (T)

B_g = Flux density in air gap (T)
H_c = Magnetic field strength in core (A/m)
H_g = Magnetic field strength in air gap (A/m)
E= Energy (J)

$$R_{mc} = \frac{l_c}{\mu_0 \mu_r A_c}$$

$$R_{mg} = \frac{l_g}{\mu_0 A_g}$$

$$\Phi = \frac{F_m}{R_m} = \frac{NI}{\frac{l_c}{\mu_0 \mu_r A_c} + \frac{l_g}{\mu_0 A_g}}$$

$$L = \frac{N^2}{R_m}$$

$$B_c = \frac{\Phi}{A_c}$$

$$B_g = \frac{\Phi}{A_g}$$

$$H_c = \frac{B_c}{\mu_0 \mu_r}$$

$$H_g = \frac{B_g}{\mu_0}$$

1.1 Energy of the magnetic field

$$E = \frac{1}{2} \oiiint BHdV = \frac{1}{2} \oiiint \frac{B^2}{\mu} dV$$

Energy density

$$E_d = \frac{B^2}{2\mu} \left[\frac{J}{m^3}\right]$$

Energy density in core:

$$E_{dc} = \frac{B_c}{2\mu_0 \mu_r}$$

Energy density in air gap:

$$E_{dg} = \frac{B_g}{2\mu_0}$$

Energy stored in core:

$$E_c = E_{dc} V_c = \frac{B_c}{2\mu_0 \mu_r} A_c l_c$$

Energy stored in air gap:

$$E_g = E_{dg} V_g = \frac{B_g}{2\mu_0} A_g l_g$$

If A_g can be assumed to be equal to A_c (field at gap does not "spread out")

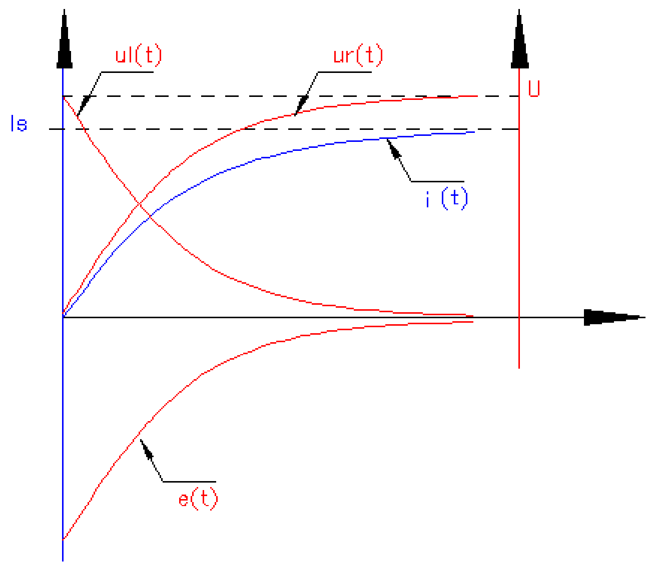
$$\frac{E_g}{E_c} = \frac{\frac{\cancel{B_g}}{\cancel{2\mu_0}} \cancel{A_g}}{\frac{\cancel{B_c}}{\cancel{2\mu_0}\mu_r} \cancel{A_c}} = \mu_r \frac{l_g}{l_c} \gg 1$$

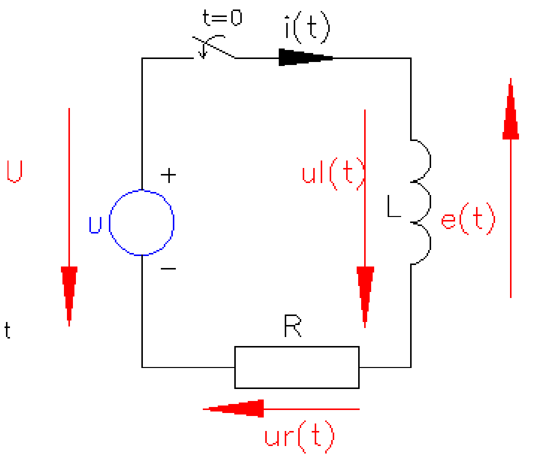
Because μ_r is large for ferromagnetic materials (thousands), most of the energy is stored in the air gap. Also, magnetic field is stronger by μ_r in the air gap (assumed flux density is constant, B_c=B_g):

$$\frac{H_g}{H_c} = \frac{\frac{\cancel{B_g}}{\cancel{\mu_0}}}{\frac{\cancel{B_c}}{\cancel{\mu_0}\mu_r}} = \mu_r$$

1.2 Energy stored in an inductance

p = Power supplied by the source
p_L= Power of inductor L
p_r = Power of resistor R
u_L= Voltage across inductor L
I_s = Stationary state current
Switch closes at t=0.





Power p supplied by the source:

$$p(t) = u(t)i(t) = p_L + p_R = u_L * i + i^2 R$$

$$u_L = L\frac{di}{dt}$$

$$p(t) = L\frac{di}{dt} i + i^2 R$$

Because

$$p = \frac{dE}{dt}$$

$$\frac{dE}{\cancel{dt}} = \frac{Li * di + i^2 R dt}{\cancel{dt}} \rightarrow dE = Li\, di\; + i^2 R dt$$

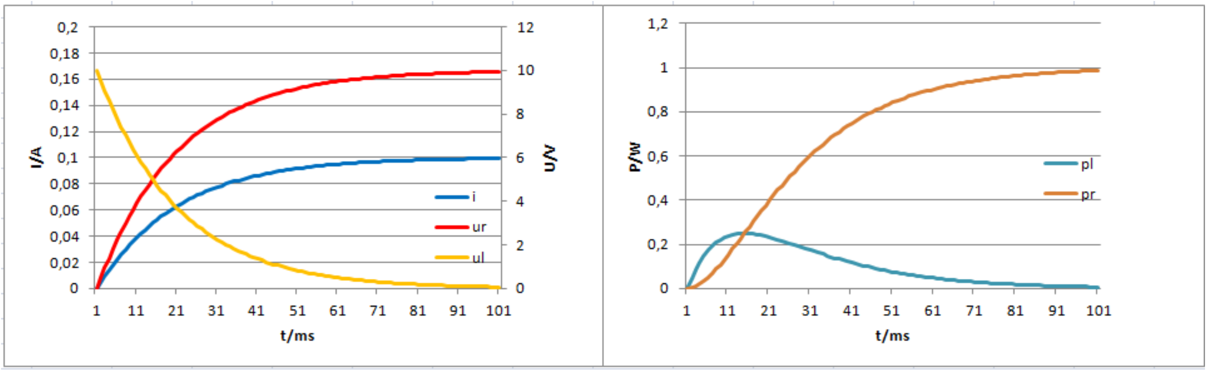
$$\int_0^E dE = \int_0^{Is} Li\, di + \int_0^t R\, i^2 dt$$

The first integral represents the energy stored in the inductance

$$\left. E \right|_0^E = \frac{1}{2} L \left. i^2 \right|_0^{Is}$$

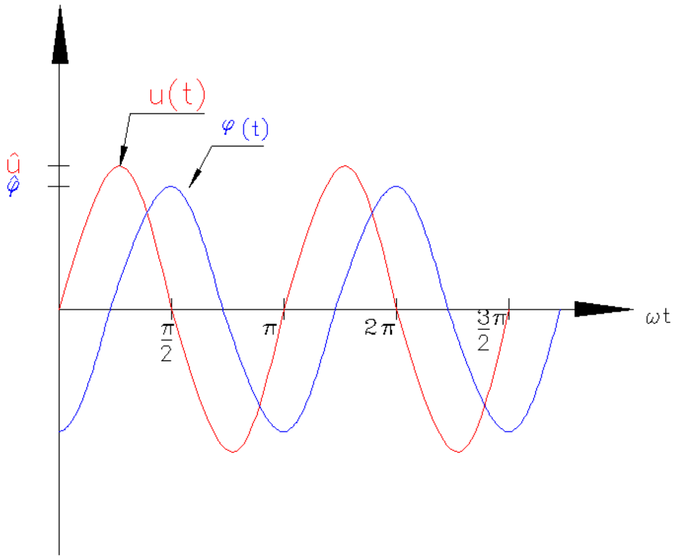
$$E = \frac{1}{2} L {I_s}^2$$

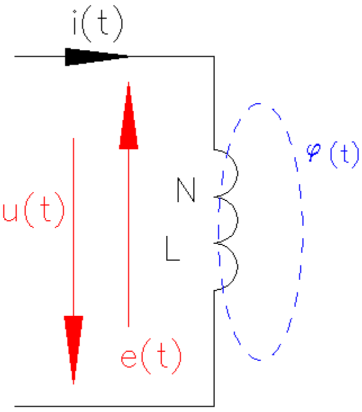
Example:
U=10V
L=2H
R=100 Ω



$$E = \frac{1}{2} * 2Vs/A \left(\frac{10V}{100V/A} \right)^2 = 0{,}01 Ws$$

1.3 EMF equation





Assume

$u(t) = -e(t) = \hat{u}\sin(\omega t),$

$e(t) = -N\dfrac{d\Phi}{dt} \rightarrow u = N\dfrac{d(\hat{\varphi}\sin(\omega t))}{dt} = N\omega\hat{\varphi}\cos(\omega t)$

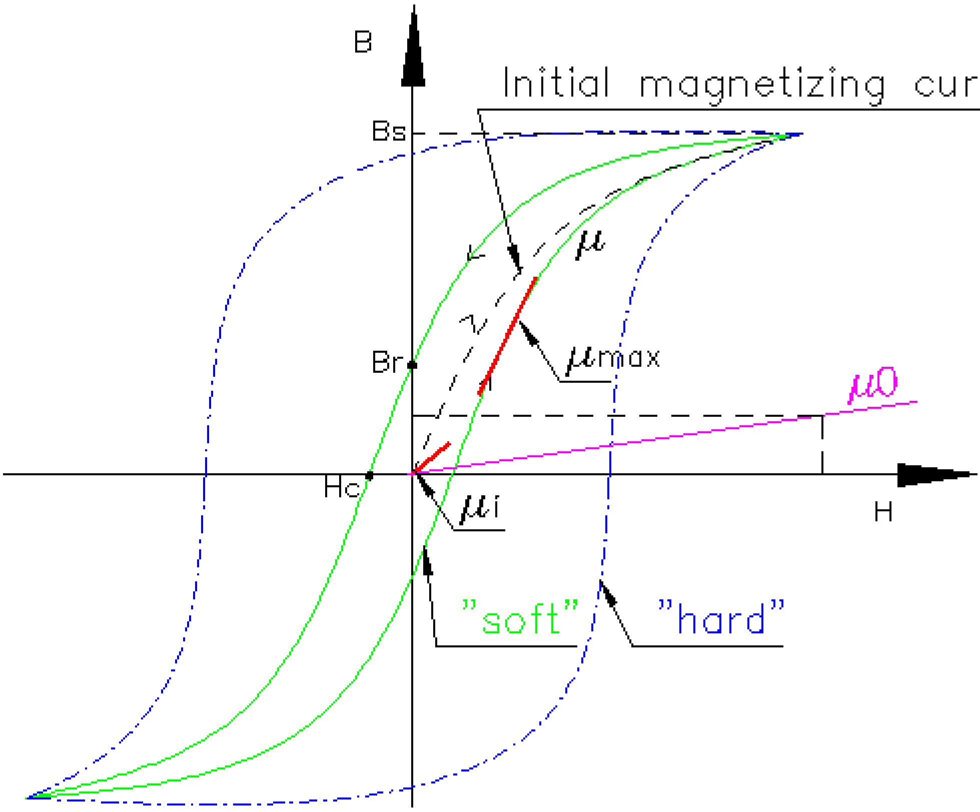
Maximum instant value of u is when $\cos(\omega t) = 1$, i.e. ωt=0;

$\hat{u} = N\omega\hat{\varphi} = 2\pi f N\hat{\varphi}$

Because u(t) is $\hat{u}\sin(\omega t)$, RMS- value of u is $U = \frac{\hat{u}}{\sqrt{2}}$

$$U = \frac{2\pi}{\sqrt{2}} f N\hat{\varphi} \approx 4{,}44 f N\hat{\varphi}$$

2 Measuring magnetic core properties



2.1 Relative permeability

Permeability of ferrous materials is not a constant. It depends on the magnetic field strength and temperature.

Definitions

a) initial permeability
The initial permeability μ_i defines the relative permeability at very low excitation levels and constitutes the most important means of comparison for soft magnetic materials. According to IEC 60401-3, μi is defined using closed magnetic circuits (e.g. a closed ring-shaped cylindrical coil) for f ≤10 kHz, B<0.25mT,T=25 °C

$$\mu_i = \frac{1}{\mu_0} \frac{\Delta B}{\Delta H}_{(\Delta H \to 0)}$$

2.1.1 Method 1

A_c= Area of cross section of the magnetic circuit (m^2)
R_m = Reluctance of the magnetic path (A/Vs)
l_m = length of magnetic circuit (m)
μ_0 = permeability of vacuum, $4\pi*10^{-7}$ H/m
μ_r = relative permeability of material (unitless)

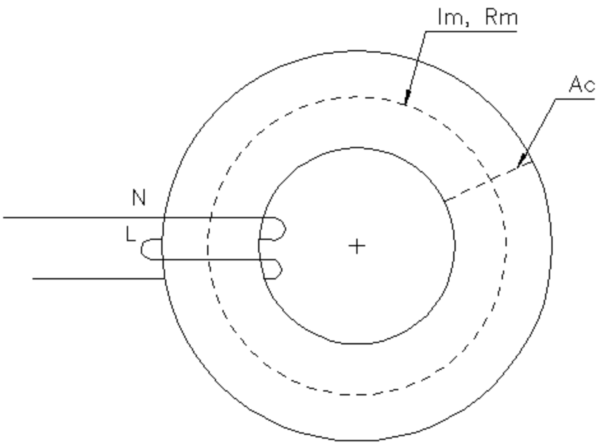


Figure 2: Relative permeability measurement principle method 1- Closed magnetic circuit

- Measure l_m and A_c (in m and m^2)- easiest in case of a toroid core
- Calculate “air core inductance” L_0 (H)

$$R_m = \frac{l_m}{\mu_0 \mu_r A_c}$$

$$L_0 = \frac{N^2}{R_{m0}} = \frac{N^2 \mu_0 A_c}{l_m}$$

- Measure (e.g. with an LCR-meter) the inductance of winding with N turns

$$L_s = \frac{N^2 \mu_0 \mu_r A_c}{l_m}$$

- Divide L_s by L_0 to get μ_r of the material in question

$$\frac{L_s}{L_0} = \frac{\frac{\cancel{N^2 \mu_0} \mu_r \cancel{A_c}}{\cancel{l_m}}}{\frac{\cancel{N^2 \mu_0 A_c}}{\cancel{l_m}}} = \mu_r$$

2.1.2 Method 2

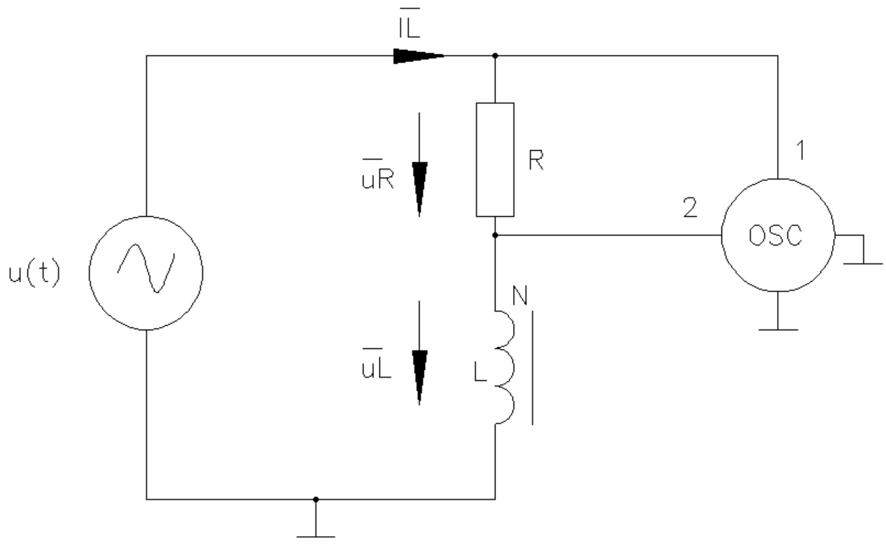


Figure 3:Relative permeability measurement principle method 2

$$\begin{cases} R_m = \dfrac{l_m}{\mu A_c} \rightarrow \boldsymbol{\mu} = \dfrac{l_m \Phi}{N I_L\, A_c} = \dfrac{l_m U}{N I_L\, A_c N \omega} = \dfrac{\boldsymbol{l_m U}}{\boldsymbol{I_L A_c N^2\ \omega}} \\ R_m = \dfrac{N I_L}{\Phi} \\ u(t) = -N \dfrac{d\Phi}{dt} \end{cases}$$

$$\frac{d\Phi}{dt} = \frac{u(t)}{-N} \rightarrow$$

$$\Phi_{ave} = \frac{1}{\pi/2} \int_0^{\pi/2} \frac{u(t)}{-N} dt = -\frac{2\hat{u}}{\pi N} \int_0^{\pi/2} \sin(\omega t)\, dt = -\frac{2\hat{u}}{\pi N \omega} \Big|_0^{\pi/2} - cos(\omega t) = \frac{2\hat{u}}{\pi N \omega}$$

Because flux is sinusoidal

$$\Phi_{ave} = \frac{2\sqrt{2} * \Phi}{\pi}$$

$$\Phi = \frac{\cancel{2}\hat{u}}{\cancel{\pi} N \omega} * \frac{\cancel{\pi}}{\cancel{2}\sqrt{2}} = \frac{\hat{u}}{\sqrt{2} N \omega} = \frac{U}{N\omega}$$

Relative permeability can be calculated

$$\mu_r = \frac{\mu}{\mu_0}$$

U is the RMS voltage of u2 in oscilloscope channel 2 (voltage over coil).

IL= (U1- U)/R

Note that voltage shall be kept low enough to avoid core saturation.

2.1.2.1 Example using methods 1 and 2

Ferrite core of Figure 7:DUT of example 1
A_c= 7,33mm x 3,33mm =2,42x10^{-5} m^2
l_m = 0,032 m
N=15
Method 1:

$$L_0 = \frac{N^2 \mu_0 A_c}{l_m} = \frac{15^2 * 4\pi * \frac{10^{-7} H}{m} * 2{,}42 * 10^{-5} m^2}{0{,}032\ m} = 213{,}7\ nH$$

Measured with an LCR meter at 1 kHz:

$$L_s = \frac{N^2 \mu_0 \mu_r A_c}{l_m} = 2{,}15\ mH$$

$$\frac{L_s}{L_0} = \mu_r = \frac{2{,}15\ mH}{213{,}7\ nH} = 10060 \approx 10000$$

Method 2:
R=18 Ω
U =U_2= 0,101V
I_L=(U_1-U_2)/R=0,125V/18 Ω =6,94 mA

$$\mu = \frac{l_m U}{I_L A_c N^2\ \omega} = \frac{0{,}032\ m * 0{,}101V}{6{,}94 * 10^{-3} A * 2{,}42 * 10^{-5} m^2 * 15^2 * 2\pi * 1 * 10^3 \frac{1}{s}} = 0{,}0135\ H/m$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0{,}0136\ H/m}{4\pi * 10^{-7} H/m} = 10843 \approx 11000$$

The result's absolute value is very sensitive on the estimation of dimensions l_m and A_c. Both methods are limited to cases where magnetic circuit is well defined and does not include an air gap.

2.2 Measuring magnetic core hysteresis loop

Area within the hysteresis loop is proportional to the losses in the core.

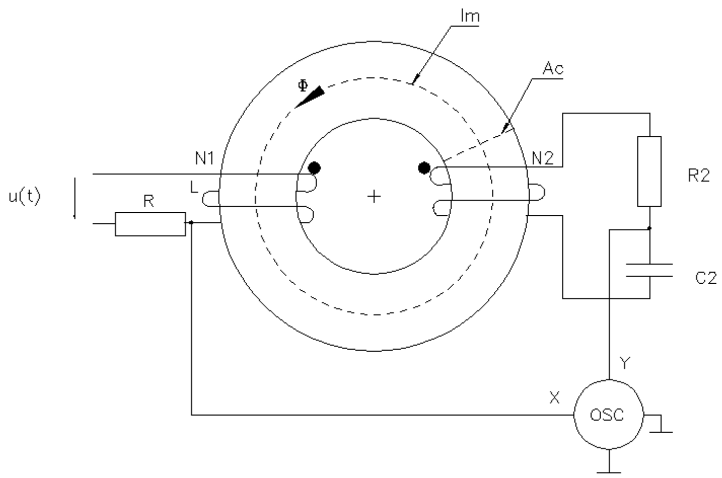


Figure 4:Measurement principle

For an inductor:

$$E = -N\frac{d\varphi}{dt} = \frac{-NAdB}{dt} \rightarrow B(t) = \int_0^t \frac{E(t)}{-NA_c}\, dt$$

$$\Phi = BA_c$$

E=u(t) =U_p sin(ωt)= Voltage across inductor (V)
N= number of turns
A_c= Area of cross section of the magnetic circuit (m^2)
Φ = Magnetic flux (Wb)
B= Flux density (T)
H= Magnetic field strength (A/m)
l_m = length of magnetic circuit (m)

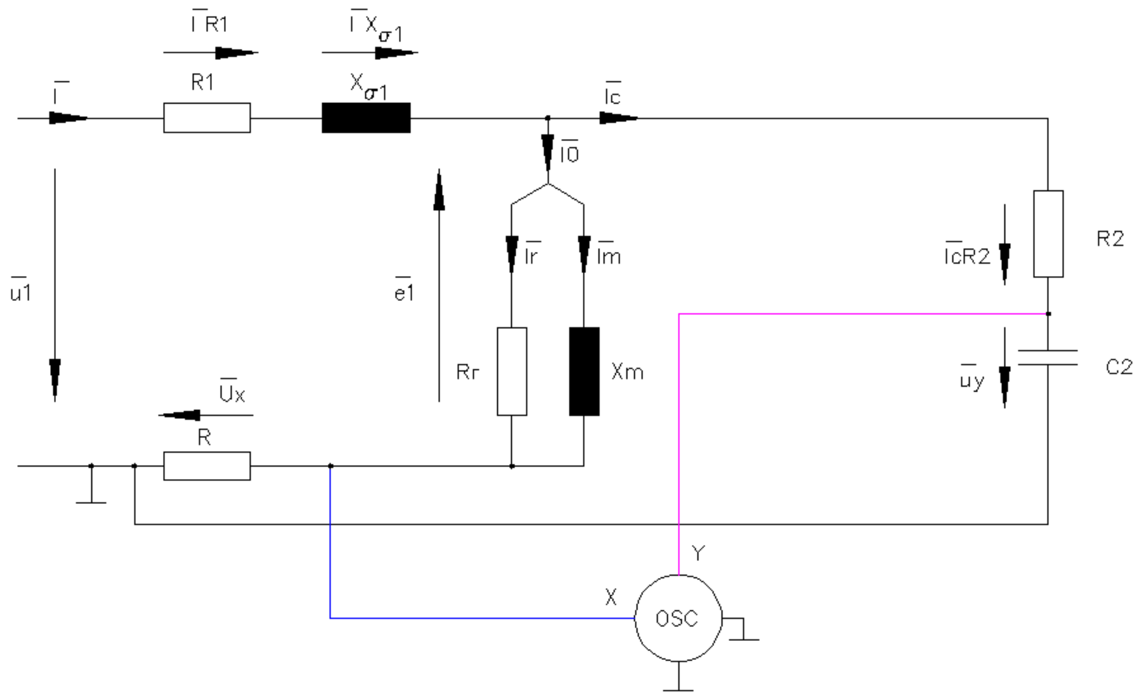


Figure 5:Equivalent circuit of device under test

R1 = Primary winding resistance
Xσ1 = Primary winding inductive stray reactance
Rr= Core iron loss resistance
Xm=Main magnetizing reactance
R2= Auxiliary resistor (R2 >>1/ωC2)
R= Auxiliary resistor (small resistance)
C2= Auxiliary capacitor, forms an integrator together with R2
Measuring hysteresis curve:

Voltage at oscilloscope horizontal plates (X) is proportional to the strength of the magnetic field H, because

$$u_x = IR$$

$$H = \frac{N_1 I}{l_m}$$

so

$$u_x = \frac{R\, l_m}{N_1}\, H = k_1 H$$

Voltage at oscilloscope vertical plates (Y) is proportional to magnetic flux density B, because if

$$R_2 \gg \frac{1}{\omega C_2}$$

And because I_r and I_c are small,

$$I \approx I_m$$

So

$$i_c \approx \frac{e_1}{R_2}$$

$$u_{C2} = u_y = i_c \frac{1}{\omega C_2} \approx \frac{e_1}{\omega R_2 C_2} \approx \frac{u_1}{\omega R_2 C_2}$$

$$u_1 \approx e_1 = N_2 \frac{d\varphi}{dt} = N_2 \frac{d[\widehat{\varphi}\sin(\omega t)]}{dt} = N_2 \widehat{\varphi}\,\omega\cos(\omega t) = \omega N_2 \varphi = \omega N_2 B A_c$$

And

$$u_y = \frac{\cancel{\omega} N_2 B A_c}{\cancel{\omega} R_2 C_2} = \frac{N_2 A_c}{R_2 C_2} B = k_2 B$$

k_1= scale factor for H
k_2= scale factor for B

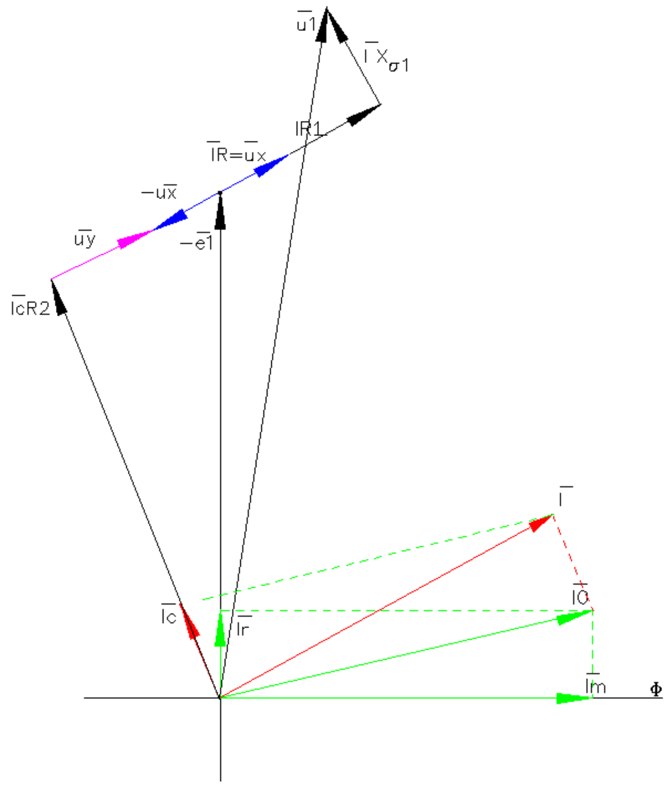


Figure 6:Vector diagram of hysteresis loop measurement (Ic and Ir are drawn exaggerated)

2.2.1 Example 1:

Toroid ferrite core, no air gap

Figure 7:DUT of example 1

Define the needed minimum number of turns N_{min} to achieve desired flux density B:

$$N_{min} = \frac{l_m\, B}{I\, \mu}$$

$\mu = \mu_0\, \mu_r$ = permeability of material (H/m)
I= Current of winding (A) (depends on the voltage source u(t) available)
$u(t) = U_p \sin(\omega t)$
N_1= 15, N_2= 15
A_c= 7,33mm x 3,33mm =$2{,}42\text{x}10^{-5}$ m^2
l_m = 0,032 m

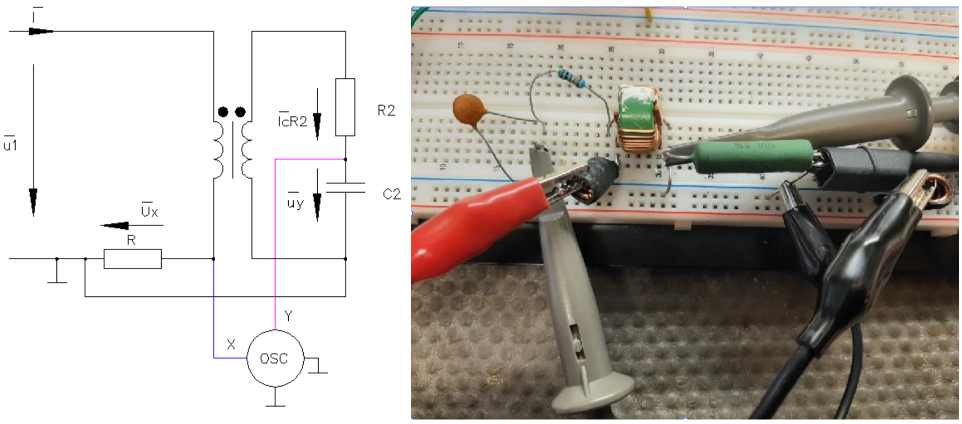


Figure 8:Schematic and test arrangement

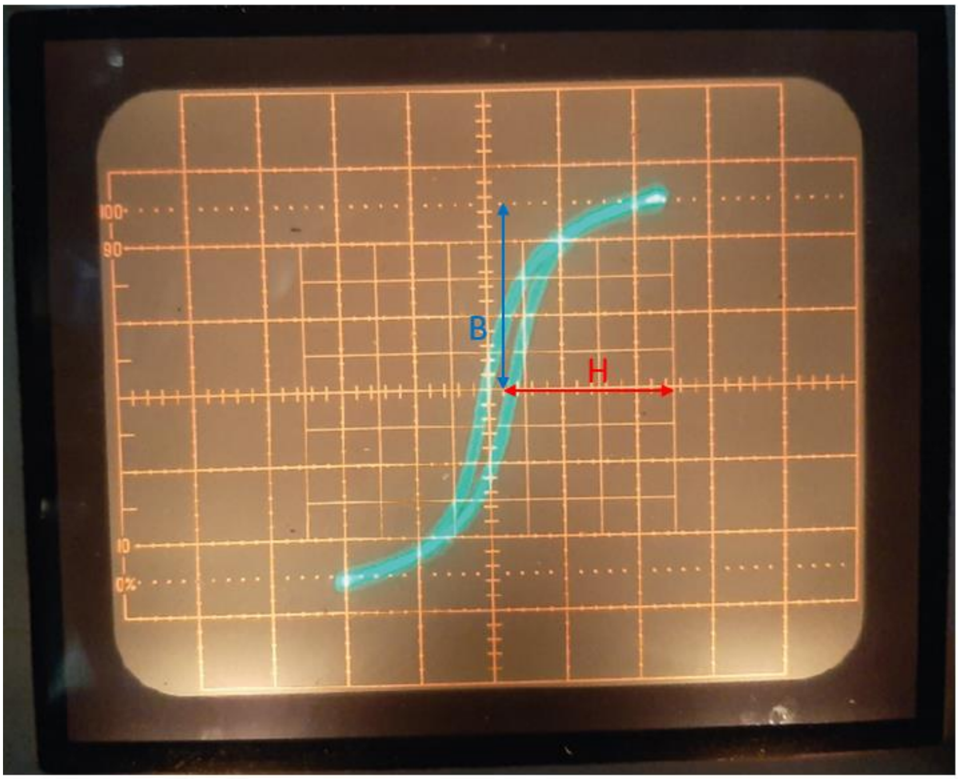


Figure 9:Hysteresis curve of DUT of example 1

X:1V/div
Y: 100mV/div
R=18Ω
R_2= 100kΩ
C_2=3,6 nF

$$k_1 = \frac{R\, l_m}{N_1} = \frac{18\text{V/A} * 0{,}032\text{ m}}{15} = 0{,}0384\, Vm/A$$

$$H = \frac{u_x}{k_1} = \frac{2{,}2\cancel{V}}{0{,}0384\,\cancel{V}m/A} \approx 57{,}3\, A/m$$

$$k_2 = \frac{N_2 A_c}{R_2 C_2} = \frac{15 * 2{,}42 * 10^{-5}\, m^2}{\frac{100 * 10^3 \cancel{V}}{\cancel{A}} * 3{,}6 * 10^{-9} \cancel{A}s/\cancel{V}} = 1{,}008 m^2/s$$

$$B = \frac{u_y}{k_2} = \frac{250 * 10^{-3} V}{1{,}008\, m^2/s} \approx 0{,}248T$$

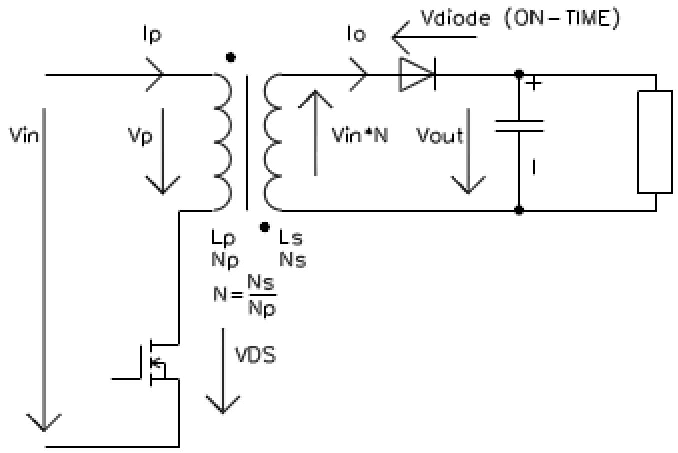
3 Switch mode power supply design example

Following calculation principle is based on "Switchmode Power Supply Handbook" by Keith Billings

Design a boost flyback converter with following general properties:

Supply voltage: 9V DC
Output voltage: 12V DC
Output current: 0,35A
Controller: LM2577

3.1 Basic topology of single ended flyback converter



I_p = Flyback transformer primary current
V_{in}= Supply voltage
V_p = Flyback transformer primary voltage
N_p, N_s = Number of turns in flyback transformer primary and secondary
L_p, L_s = Inductance of flyback transformer's primary and secondary
I_o= Flyback transformer secondary current
V_{out} = Output voltage
V_{DS}=Mosfet switch drain to source voltage
V_{dr}= Rectifier diode reverse voltage
V_{df}= Rectifier Diode forward voltage drop
T= Switch frequency period
A_e=Cross sectional area of transformer core

Initial design parameters

Parameter		Value	Unit
Input voltage	V_{in}	7,5-9	V_{DC}
Output power	Pout	4,2	W
Output voltage	Vout	12	V
Efficiency	η	85	%
Rectifier Diode forward voltage drop	V_{df}	0,7	V
Switching frequency	f	52	kHz

3.2 Select core cross section

Initially only the cross section of the core will be needed for calculation of primary minimum turns. Select A_e=64 mm^2. Core material property dependent A_L and required air gap l_g will be determined later.

3.3 Select maximum on-time

$$T = \frac{1}{f} = \frac{1}{52 * 10^3 Hz} = 19{,}2\mu s$$

Maximum on-time will occur at minimum input voltage and maximum load. Here it is assumed that maximum on-time can not exceed 50% of the total period.

$$T_{ON(max)} = \frac{19{,}2}{2}\mu s \approx 9{,}6\mu s$$

To allow some margin for control to operate well at minimum input voltage, let's select

$$T_{ON(max)} = 9{,}0\mu s$$

3.4 Define minimum input voltage

$V_{in(min)=}$7,5V

3.5 Select working flux density swing

ΔB_{ac}=200 mT (Peak-to-Peak value)

3.6 Calculate minimum primary turns

$$Np_{min} = \frac{V_{in(min)} T_{ON(max)}}{\Delta B_{ac} A_e} = \frac{7{,}5\cancel{V} * 9{,}6 * 10^{-6}\cancel{s}}{200 * 10^{-3}\frac{\cancel{Vs}}{\cancel{m^2}} * 64 * 10^{-6}\cancel{m^2}} = 5{,}6 \approx 6$$

3.7 Calculate secondary turns

Calculate volts per turn in primary

$$\frac{V_p}{turn} = \frac{7{,}5V}{6} = 1{,}25V/turn$$

Secondary turns rounded up to nearest integer

$$N_s = \frac{V_{out} + V_{df}}{V/turn} = \frac{12V + 0{,}7V}{1{,}25\ V} = 10{,}2 \approx 11$$

Volts per turn in the secondary would actually be

$$\frac{V_s}{turn} = \frac{12V + 0{,}7V}{11} = 1{,}15V/turn$$

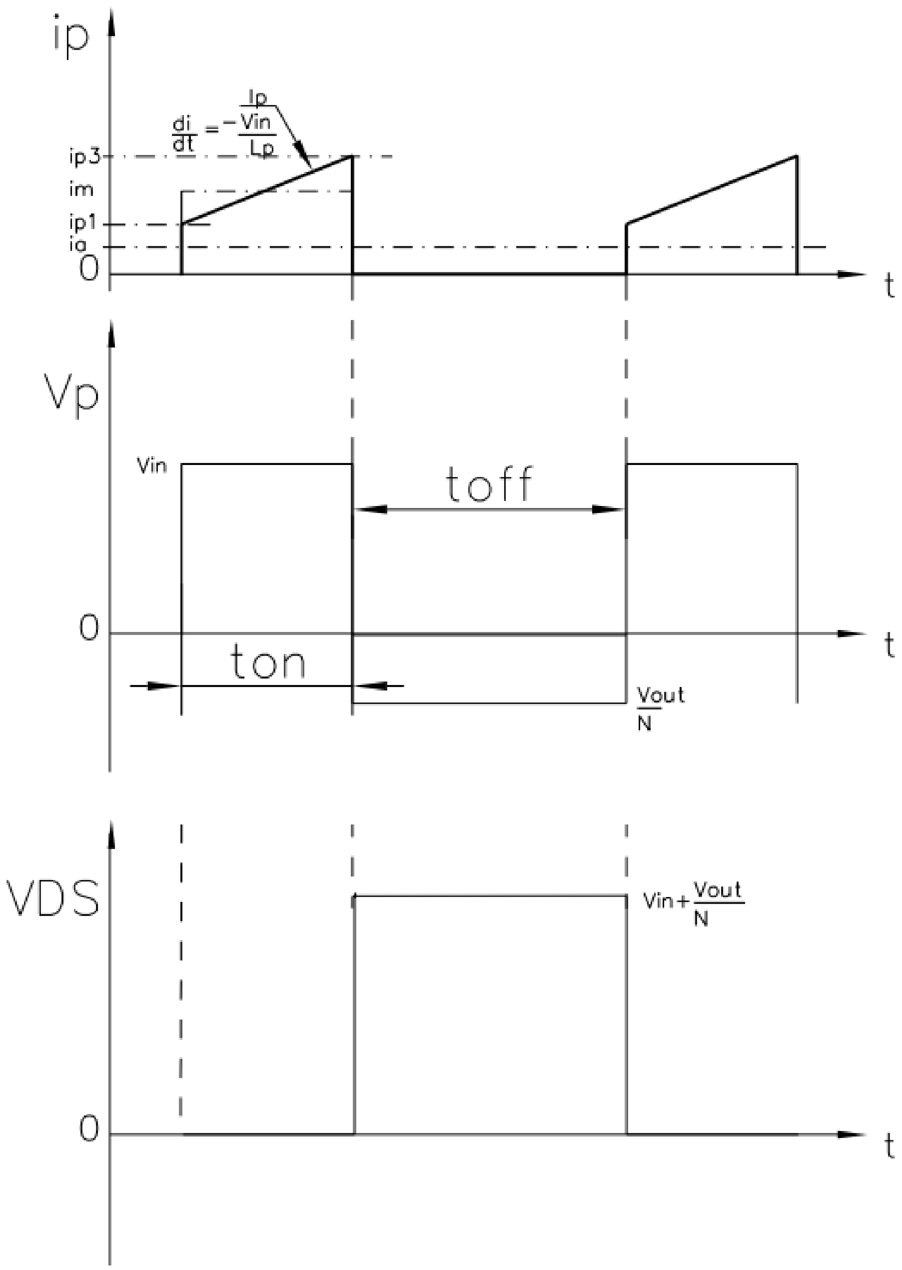
Due to secondary turns roundoff, T_{on} and T_{off} must be adjusted to maintain volt-second equality:

$$\frac{V_p T_{on}}{N_p} = \frac{V_s T_{off}}{N_s} \rightarrow T_{on} = \frac{N_p V_s\ T}{N_s V_p + N_p V_s} = \frac{6 * 12{,}7V * 19{,}2\mu s}{11 * 7{,}5V + 6 * 12{,}7V} = 9{,}2\mu s$$

3.8 Calculate primary inductance and core air gap size

I_a= Average current during period T
I_m= Mean current during T_{on}

Incomplete energy transfer mode with optimum primary inductance



$$V_{in} = L_p \frac{\Delta I_p}{\Delta t}$$

If by choice

$$i_{p3} = 3i_{p1} \rightarrow i_m = 2i_{p1}$$

Average current I_a during the whole period T can be calculated

$$I_a = \frac{P_{out}}{\eta\, V_{in}} = \frac{4{,}2W}{0{,}85 * 7{,}5V} = 0{,}66A$$

Therefore, mean current during time T_{on} is

$$I_m = \frac{I_a T}{T_{on}} = \frac{0{,}66A * 19{,}2\mu s}{9{,}2\mu s} = 1{,}38A$$

Change of current I_p during the on-period is $2i_{p1}=I_m$. Inductance of the primary can now be calculated:

$$L_p = \frac{V_{in}\Delta t}{\Delta i} = \frac{7{,}5V * 9{,}2\mu s}{1{,}38A} = 50\mu H$$

Assuming that all reluctance R_m is in the air gap l_g and the field will not spread out significantly in the air gap i.e. $A_e=A_g$:

$$R_m = \frac{l_c}{\mu_0\mu_r A_e} + \frac{l_g}{\mu_0 A_g} \approx \frac{l_g}{\mu_0 A_e}$$

and

$$L_p = \frac{{N_p}^2}{R_m} = \frac{{N_p}^2}{\frac{l_g}{\mu_0 A_e}} \rightarrow l_g = \frac{\mu_0 {N_p}^2 A_e}{L_p}$$

air gap l_g

$$l_g = \frac{\mu_0 {N_p}^2 A_e}{L_p}\left[\frac{\frac{\cancel{Vs}}{\cancel{Am}}m^{\cancel{2}}}{\frac{\cancel{Vs}}{\cancel{A}}} = m\right] = \frac{4\pi * 10^{-7} * 6^2 * 64 * 10^{-6}}{50 * 10^{-6}} m = 0{,}058mm$$

$\mu_0=4\pi*10^{-7}$ H/m

3.9 Final specification of the core

Typically, the AL value, also known as AL factor, inductance factor, inductance coefficient, inductance per turn or inductance per square turn is given in the core's datasheet for gapped and non-gapped cores. An air gap closest to calculated in previous phase is selected.

$$A_L = \frac{L_p}{{N_p}^2}$$

RM8				
Material	AL/nH/N^2	Air gap/mm	Np(min)	Lp/µH
N48	630	0,10	6	22,6
N48	400	0,14	6	14,4
N48	**315**	**0,17**	**6->13**	**11,3->53,2**

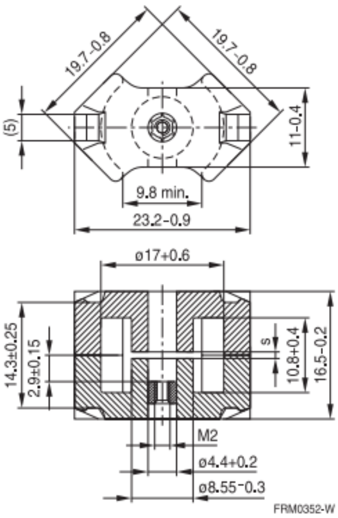
As in this example, if the core material N48 was gapped 0,17mm, it turns out that the number of primary turns must be increased from 6 to 13 to achieve the required minimum primary inductance. Recalculation according to 3.7 and 3.8 must be in that case performed.

Alternatively, material with higher AL with a suitable air gap providing at least the minimum inductance required for the primary could be selected and keeping the primary turns as they were.

RM 8

Core **B65811**

- To IEC 62317-4
- Cores without center hole for transformer applications
- Delivery mode: sets



Magnetic characteristics (per set)

	with center hole	without center hole	
Σl/A	0.68	0.59	mm^{-1}
l_e	35.1	38	mm
A_e	52	64	mm^2
A_{min}	—	55	mm^2
V_e	1825	2430	mm^3

Approx. weight (per set)

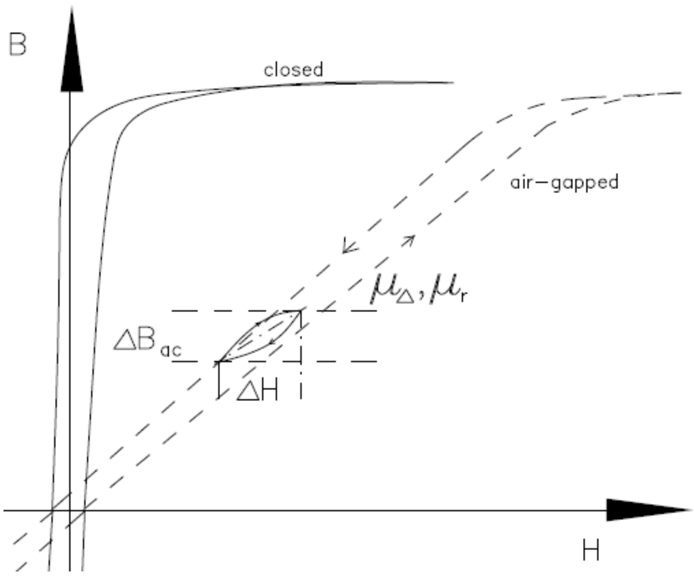
m	10.7	12	g

Gapped

Material	A_L value nH	s approx. mm	μ_e	Ordering code [1)] -D with center hole -F with threaded sleeve -J without center hole
N48	250 ± 3%	0.23	134	B65811+0250A048
	315 ± 3%	0.17	169	B65811+0315A048
	400 ± 3%	0.14	215	B65811+0400A048
	630 ± 5%	0.10	338	B65811+0630J048
N41	160 ± 3%	0.49	76	B65811J0160A041
	250 ± 5%	0.24	118	B65811J0250J041
	630 ± 5%	0.11	298	B65811J0630J041
	1600 ±10%	0.04	756	B65811J1600K041
N87	250 ± 3%	0.30	118	B65811J0250A087
	400 ± 3%	0.18	189	B65811J0400A087

3.10 Calculate flux density swing AC component

$$B_{ac(p-p)} = \frac{V_{in(\min)} * T}{N_p * A_e} = \frac{7,5V * 19,2 * 10^{-6}s}{13 * 64 * 10^{-6}m^2} = 173mT$$



3.11 Calculate contribution of the flux DC component

Using an equation for solenoid magnetic field in the air gap with average primary DC current I_a

$$B_{DC} = \frac{\mu_0 N_p I_a}{l_g} = \frac{4\pi * 10^{-7} \frac{Vs}{Am} * 13 * 0{,}66A}{0{,}17 * 10^{-3} m} = 63\, mT$$

$$B_{max} = B_{ac} + B_{DC} = 173mT + 63mT = 237mT$$

3.12 Check B_{max} against core material flux density characteristics at 100 °C to leave sufficient margin for saturation.

SIFERRIT materials

N48

Material properties

Preferred application			Resonant circuit inductors
Material			N48
Base material			MnZn
Color code (adjuster)			—
	Symbol	Unit	
Initial permeability (T = 25 °C)	μ_i		2300 ±25%
Meas. field strength Flux density (near saturation) (f = 10 kHz)	H B_S (25 °C) B_S (100 °C)	A/m mT mT	1200 420 310
Coercive field strength (f = 10 kHz)	H_c (25 °C) H_c (100 °C)	A/m A/m	26 19
Optimum frequency range	f_{min} f_{max}	MHz MHz	0.01 0.1
Relative loss factor at f_{min} / at f_{max}	$\tan\delta/\mu_i$	10^{-6} 10^{-6}	<4 <6
Hysteresis material constant	η_B	10^{-6}/mT	<0.4
Curie temperature	T_C	°C	>170
Relative temperature coefficient at 25 … 55 °C at 5 … 25 °C	α_F	10^{-6}/K	 0.3 … 1.3 0.3 … 1.3
Mean value of α_F at 25 … 55 °C		10^{-6}/K	0.70
Density (typical values)		kg/m³	4700
Disaccommodation factor at 25 °C	DF	10^{-6}	2
Resistivity	ρ	Ωm	3
Core shapes			RM, P